MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 1, 2017/2018

TMA1201 – DISCRETE STRUCTURES AND PROBABILITY

(All sections / Groups)

23 OCTOBER 2017
9:00 am –11:00 am
(2 Hours)

INSTRUCTION TO STUDENT

1. This question paper consists of 7 printed pages (inclusive of the front page) with 3 questions only. Pages 5 to 7 are appendices for the rules of inference, logical equivalence laws and standard normal distribution table.
2. Attempt **ALL 3** questions. The distribution of the marks for each question is given.
3. Please write all your answers in the answer booklet provided.
4. Show **ALL** of your working steps clearly.
5. **EXCEPT Questions 3b and 3c**, calculators are **ALLOWED** in this examination.

QUESTION 1 (17 MARKS)

a) Let $A=\{x \mid x=k^2 \text{ where } k \in N \wedge x \leq 20\}$ and $B=\{x \in Z^+ \mid x \text{ is an odd} \wedge x \leq 9\}$.

i) List the elements in A and B. [1 mark]

ii) Find $A-B$ and $P(A-B)$ [2 marks]

iii) Is $A-B \subseteq P(A-B)$? Justify your answer. [1 mark]

b) Let R be the relation defined on the set $A=\{3,5,6,12\}$, where

$R=\{(a,b) \in A \times A \mid |a-b| \text{ is a prime number}\}$.

i) List all elements in R. [1 mark]

ii) Is R symmetric? Justify your answer. [1 mark]

iii) Is R is transitive? Justify your answer. [1 mark]

iv) Is R a function? Justify your answer. [1 mark]

c) By constructing a TRUTH TABLE, determine whether the statement $(Q \wedge ((P \wedge Q) \vee (\neg P \wedge \neg Q))) \rightarrow P$ is a tautology, contingency or contradiction. [2 marks]

d) Consider the following predicates with the universe of discourse (UOD) consist of all people:

$A(x)$: x is generous.

$B(x)$: x is happy.

$C(x)$: x is intelligent.

$D(x)$: x is very free with his money.

Given the following argument:

Generous people are happy. Aladin is intelligent, but not happy. Everyone is either generous or they are not very free with their money. Hence, someone is intelligent but not very free with his money.

i) Translate the argument into THREE premises and ONE conclusion. [2 marks]

ii) Show that the translated argument in i) is valid with formal reasoning. Please present all your steps and state clearly each of the laws that you have used in your proof. [5 marks]

Continued .....

QUESTION 2 (16 MARKS)

a) Use mathematical induction to prove that for all integers $n \geq 1$,

$$1+3+5+...+(2n-1)=n^2.$$ [5 marks]

b) Show that $f(n)=7n^2+n\ln n$ is $\Theta(n^2)$ for all integers $n>1$. [3 marks]

c) Let A be the finite-state automaton with sets of finite states $S=\{S_1, S_2, S_3\}$ and input values $I=\{0,1\}$. The states S_1 and S_3 are its initial and accepting states, respectively. The next-state function is given as follows.

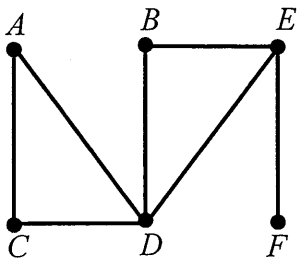
State	Next state	
	0	1
S_1	S_2	S_1
S_2	S_3	S_2
S_3	S_3	S_3

i) Construct the state-transition diagram for the automaton A. [1 mark]

ii) Give a bit string of length four that is accepted by A. Give a bit string of length four that is not accepted by A. [1 mark]

iii) Use regular expression to express the language that is accepted by the automaton A. [1 mark]

d) Given the following graph:



Use the breath first search algorithm to find a spanning tree in the above graph. Assume vertex B is the root and the vertices are ordered alphabetically. Show clearly each step of how the algorithm is performed and present your answer in a table with three columns, the first column is the number of steps, the second column is the queue for the search algorithm and the third column is the edges of the resulting spanning tree. [5 marks]

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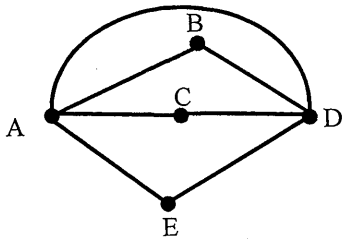
QUESTION 3 (17 MARKS)

a) Determine whether the following graph has an Euler circuit. Construct such a circuit when one exists. [2 marks]



[IMPORTANT: CALCULATORS ARE NOT ALLOWED IN QUESTIONS 3b AND 3c. YOU MUST SHOW PROPER STEPS ON HOW TO SIMPLIFY AND COMPUTE THOSE ANSWERS.]

b) In a club of 10 members,

i) how many ways are there to choose three members of the club to serve on an executive committee? [1.5 marks]

ii) how many ways are there to choose a president, a secretary, and a treasurer of the club, where no person can hold more than one position? [1.5 marks]

c) How many different strings can be made from the letters in SESAMES, using all the letters? [2 marks]

[Note: round the answers in 3d, 3e, and 3f to 4 decimal places.]

d) Suppose that 6% of all soccer players use steroids. A soccer player who uses steroids is tested positive for steroids 96% of the time, and a soccer player who does not use steroids is tested positive for steroids 5% of the time. What is the probability that a randomly selected soccer player who tests positive for steroids actually uses steroids? [3 marks]

e) Assume that the probability of a child is a girl is 0.55 and that the sexes of children born into a family are independent. What is the probability that a family of four children has

i) exactly three girls? [2 marks]

ii) at least one boy? [2 marks]

f) A machine that produces steel-balls has initially been set so that the true average diameter of the ball it produces is 3.00 cm. A steel-ball is acceptable if its diameter is within 0.04 cm of this target value. Suppose, however, that the setting has changed during the course of production, so that the steel-balls have diameters which are normally distributed with mean value 2.99 cm and standard deviation 0.02 cm. What percentage of the steel-ball produced will not be acceptable? [3 marks]

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Appendix I

List of Rules of Inference

Addition : $p \Rightarrow (p \vee q)$

Conjunction : $p \wedge q \Rightarrow (p \wedge q)$

Disjunctive Syllogism: $[(p \vee q) \wedge (\neg p)] \Rightarrow q$

Hypothetical Syllogism: $[(p \rightarrow q) \wedge (q \rightarrow r)] \Rightarrow (p \rightarrow r)$

Modus Ponens: $[(p \rightarrow q) \wedge p] \Rightarrow q$

Modus Tollens : $[(p \rightarrow q) \wedge (\neg q)] \Rightarrow \neg p$

Resolution: $[(p \vee q) \wedge (\neg p \vee r)] \Rightarrow (q \vee r)$

Simplification: $[p \wedge q] \Rightarrow p$

List of Logical Equivalence Laws

Associative : (i) $p \vee (q \vee r) \Leftrightarrow (p \vee q) \vee r \Leftrightarrow p \vee q \vee r$

(ii) $p \wedge (q \wedge r) \Leftrightarrow (p \wedge q) \wedge r \Leftrightarrow p \wedge q \wedge r$

Commutative : (i) $p \wedge q \Leftrightarrow q \wedge p$ (ii) $p \vee q \Leftrightarrow q \vee p$

Conversion of Implication: $p \rightarrow q \Leftrightarrow \neg p \vee q$

Distributive : (i) $p \wedge (q \vee r) \Leftrightarrow (p \wedge q) \vee (p \wedge r)$

(ii) $p \vee (q \wedge r) \Leftrightarrow (p \vee q) \wedge (p \vee r)$

De Morgan : (i) $\neg(p \wedge q) \Leftrightarrow (\neg p \vee \neg q)$

(ii) $\neg(p \vee q) \Leftrightarrow (\neg p \wedge \neg q)$

Domination : (i) $p \wedge F \Leftrightarrow F$ (ii) $p \vee T \Leftrightarrow T$

Double Negation: $\neg \neg p \Leftrightarrow p$

Idempotent : (i) $p \vee p \Leftrightarrow p$ (ii) $p \wedge p \Leftrightarrow p$

Identity : (i) $p \wedge T \Leftrightarrow p$ (ii) $p \vee F \Leftrightarrow p$

Negation : (i) $p \wedge \neg p \Leftrightarrow F$ (ii) $p \vee \neg p \Leftrightarrow T$

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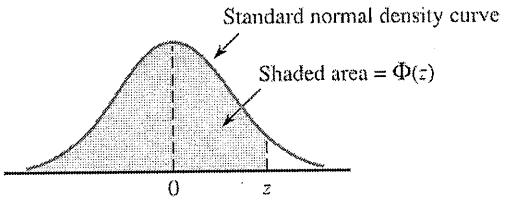
Appendix II Standard Normal Curve Areas

Standard Normal Curve Areas

$\Phi(z) = P(Z \leq z)$



z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0017	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0038
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0352	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0722	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3482
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

Continued .....

Standard Normal Curve Areas (Cont.) $\Phi(z) = P(Z \leq z)$

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9278	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

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